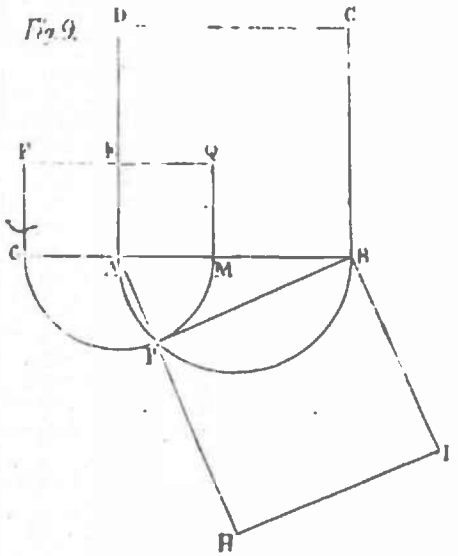
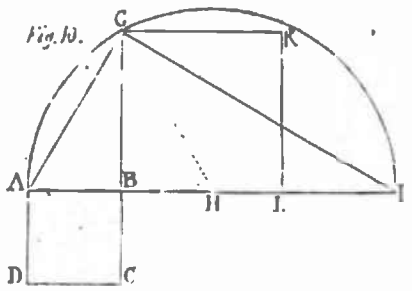
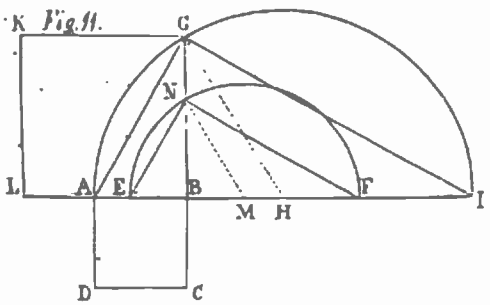
GEOMETRY OF THE SQUARE.







CLOCK TOWER AND ENGINE-HOUSE, EPSOM.

On August 2nd, an advertisement appeared in our columns inviting architects to submit designs for the above. From those which were sent in the committee unanimously adopted one by Messrs. Butler and Hodge, of London, an engraving of which we give above.

The extreme height of the tower is about 70 feet, and the openings for the dials are 4 feet 6 inches in diameter. It is to be built of red and Suffolk bricks, the external faces rubbed and finished fairly. The four lions, each bearing shields charged with the arms of the principal noblemen in the locality, and the carved portions, pedestals, &c., are to be of Caen stone. The polychromatic effect of the materials and general proportions promise to make it a conspicuous and agreeable object.

We did not see the other designs.

GEOMETRY OF THE SQUARE.*

PROBLEM.—*To construct a square that shall be equal in area to the difference between two given squares; that is, to subtract one given square from another, and to convert the difference into a square.*

Example.—Let ABCD and AEFG (fig. 9) be the given squares; it is required to subtract the lesser square, AEFG, from the greater, ABCD, and to convert the difference into a square.

Construction.—Place the squares in immediate juxta-position with a side of each in the same straight lines, and upon AE, the side of the lesser square and within the greater, describe the square AEQM, and the intermediate space BCDEQM will be the difference between the given squares, and that which the problem requires to be converted into a square.

Upon AB, the side of the larger square, describe the semi-circle APB, in which, from the point A, inflect the straight line AP, equal to AG or AM, the side of the lesser square, and join BP; then is BP the side of a square which will contain the same area as the intermediate space BCDEQM. On BP describe the square BPHI, and the thing required by the problem is done, for the square BPHI is equal in area to the space BCDEQM contained between the squares ABCD and AEQM.

Demonstration.—Because the angle APB is the angle in a semi-circle, it is a right angle, and, consequently, by the property of the right-angled triangle, the square described upon the side AB, which subtends the right angle, is equal to the sum of the squares of the sides AP and BP containing it; but AP is equal to AM, and, therefore, the square of AB is equal to the sum of the squares of AM and BP; that is, the square ABCD is equal to both the squares AEQM and BPHI; from each of these equals take away the square AEQM, and the remainder BCDEQM will be equal to the square BPHI; consequently, the difference between two given squares has been converted into a square of equivalent area, as required by the conditions of the problem.

PROBLEM.—*To construct a square that shall be equal in area to any proposed multiple, or any number of times a given square; that is, to convert a given square into another that shall contain its area any proposed number of times.*

Example.—Let ABCD (fig. 10) be the given square, of which the side is AB; it is required to convert the square ABCD into another that shall contain its area any given number of times—say three times.

Construction.—Produce AB, the side of the given square, directly forward to I, making BI equal to three times AB, or any other number of times AB, by which the given square is proposed to be multiplied. Bisect AI in H, and on H, as a centre, with the radius HA or HI, describe the semicircle AGI, meeting CB, the side of the square produced in G; then is BG the side of the required square. Upon BG, the side of the square thus found, construct the square BGKL, and it will contain an area equal to three times the given square ABCD.

Demonstration.—We have to prove that BG^2 is equal to $3\,AB^2$; and for this purpose draw the straight lines AG and IG; then, because the angle AGI is an angle in a semicircle, it is a right angle; and since GB is perpendicular to the diameter AI, the triangles AGB and GBI are similar to one another, and, consequently, their corresponding sides are proportional; that is, AB : BG :: BG : BI; but BI is equal to three times AB by the construction; therefore we get AB : BG :: BG : 3AB; and by comparing the products of the extreme and mean terms, we obtain $BG^2 = 3\,AB^2$, so that the truth of the construction is manifest.

Note.—There is another mode of finding a square, that shall be any proposed multiple of a given square, in very common use among practical men; but this method is greatly superior in point of facility, as we can obtain any proposed multiple directly, without having first determined the intermediate multiples; it is therefore recommended as being more convenient in practice when the multiple is a high number, and also as involving a principle which is not apparent by the other method.

PROBLEM.—*To construct a square that shall be any sub-multiple of a given square; that is, to divide a given square into any proposed number of equal parts.*

Example.—Let BGKL (fig. 11) be the given square of which the containing sides are BG and BL; it is required to divide the given square BGKL into any number of equal parts—*say three equal parts.*

Construction.—On BL, the side of the given square, take BE of any magnitude at pleasure, either greater or less than BL; produce LB to I, and on BI set off as many times BE as the parts into which it is proposed to divide the square; in the present example three parts. Now since BE has been taken less than BL, the side of the given square, three times BE set off towards I will reach to F: on M as a centre, and with FE as a diameter, describe the semicircle ENF intersecting BG the side of the given square in N, and draw NE, NF; then will BE be the side of a square which contains one-third of the square upon BN. Join NE and NF, and through the point G draw GA and GI respectively, parallel to NE and NF, and meeting the straight line LI in the points A and I; then shall AB be the side of the square sought, and ABCD the required submultiple of BGKL.

Demonstration.—By the principle employed in the construction of the last problem, BN is the side of a square containing three times the area of the square upon BE. But the triangles ENB and AGB are similar; therefore it is, as BN : BG :: BE : BA; and for the

* See pages 350 and 373, ante.

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